Using the Topology of Large Scale Structure as a Cosmological Probe

Stephen Appleby
Collaborators – Changbom Park, Juhan Kim, Sungwook Hong

SAA, C.B. Park, S.W. Hong, J.H. Kim
(ApJ 836, 45) (2017)
SAA, C.B. Park, S.W. Hong, J.H. Kim
(ApJ accepted) (2018)

7th Survey Science Group Workshop,
15th - 17th January 2018

Overview

KOREA INSTITUTE FOR ADVANCED STUDY

- Introduction – what is the genus of a two-dimensional field?
- How can we extract cosmological information from the genus amplitude?
- Systematic effects – RSD and shot noise
- Cosmological parameter constraints – proof of concept using mock galaxies

Genus - Definition

KOREA INSTITUTE FOR ADVANCED STUDY

- Dark matter can be described as an initially Gaussian three dimensional field – our goal is to extract cosmological information from the dark matter field in the low redshift Universe, which is traced by galaxies.

- We study two dimensional slices of the three dimensional density field.

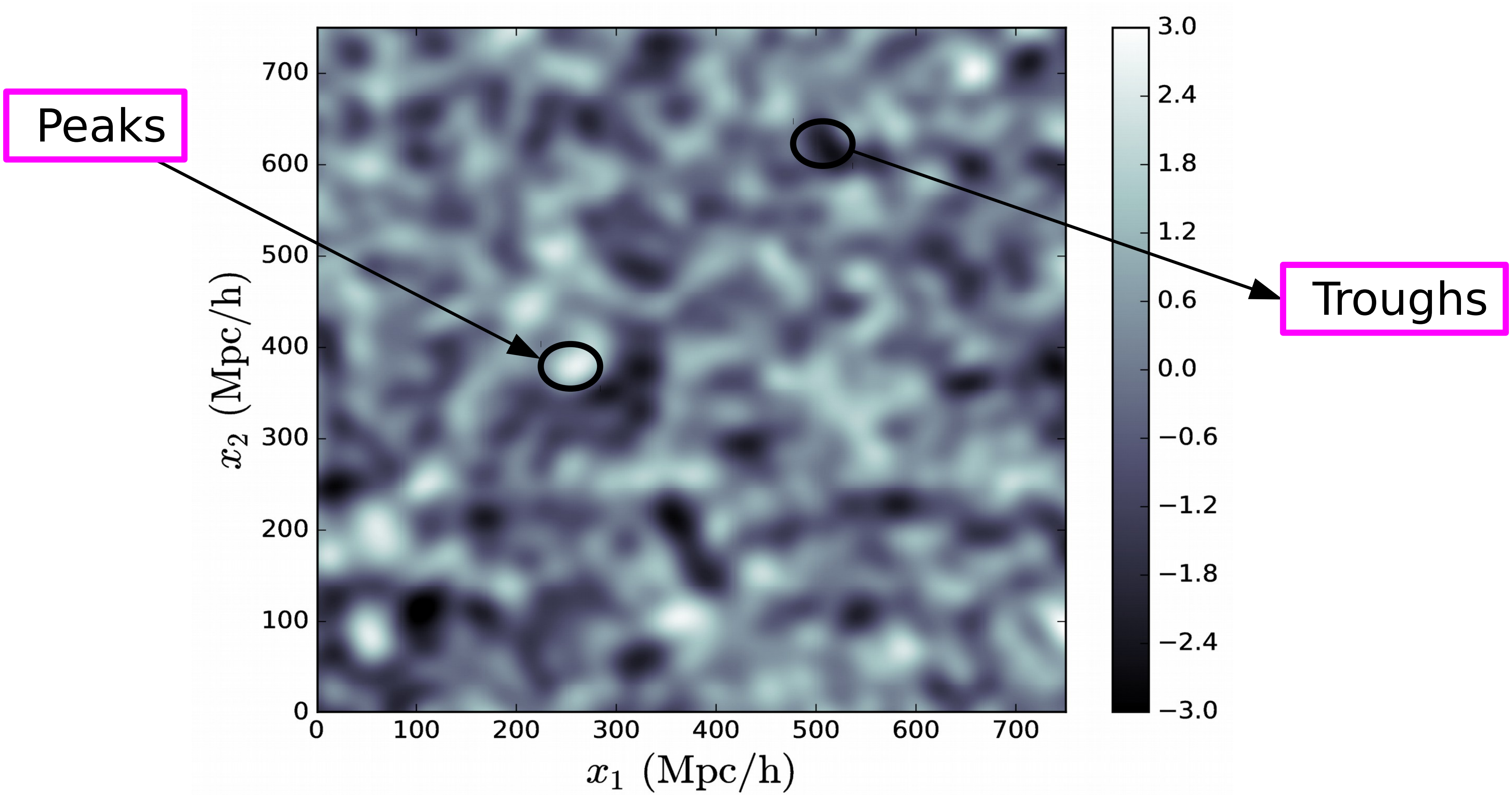
- The statistic that we use is the genus, which is a topological quantity. It is independent of morphology

- For a two dimensional cosmological field, we can define the genus in a very simple way

Genus = number of connected regions – number of holes

Genus of a Two-Dimensional Field

- Example – two-dimensional slice of a three-dimensional Gaussian Random Field

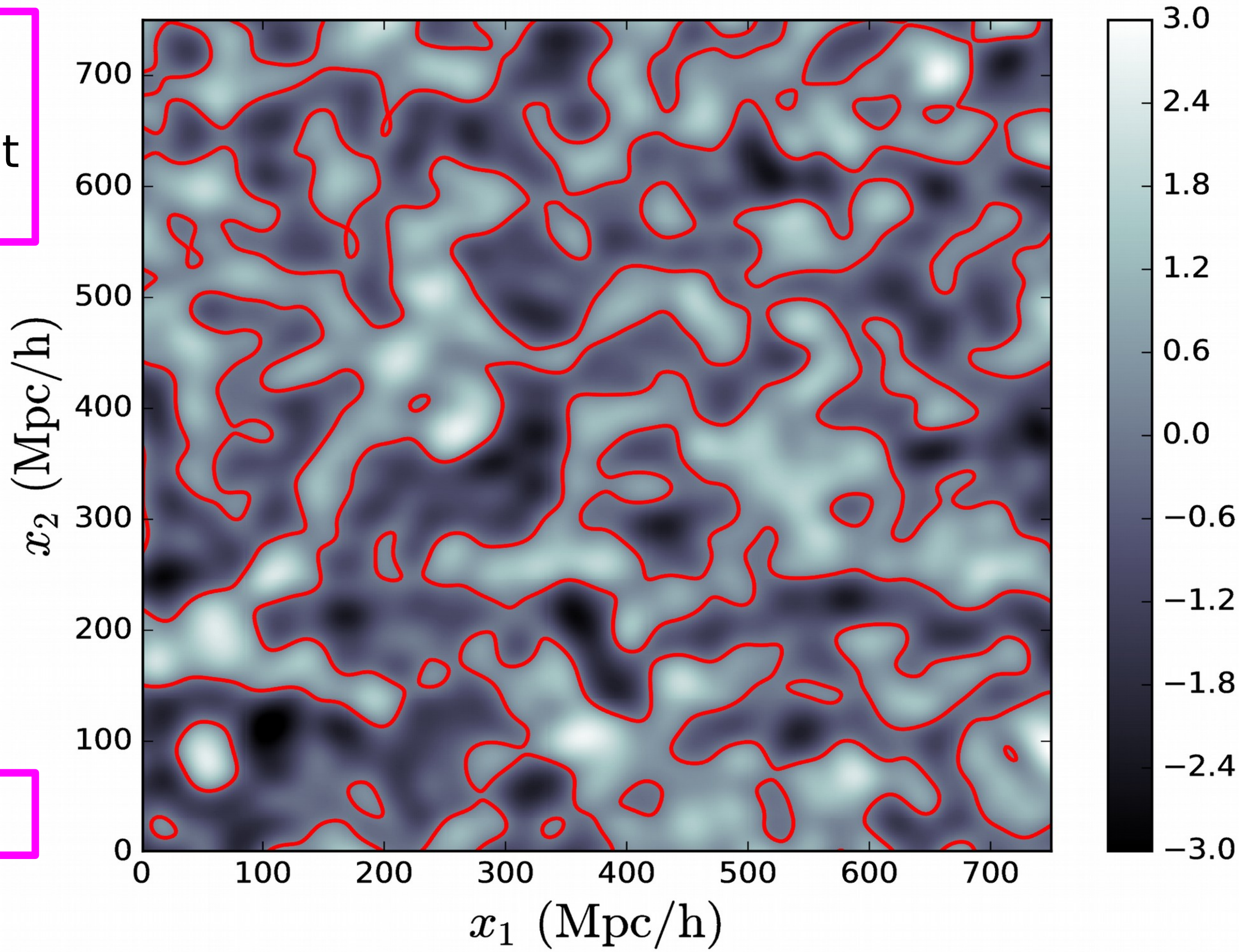


Genus of a Two-Dimensional Field

Genus = number of connected regions – number of holes

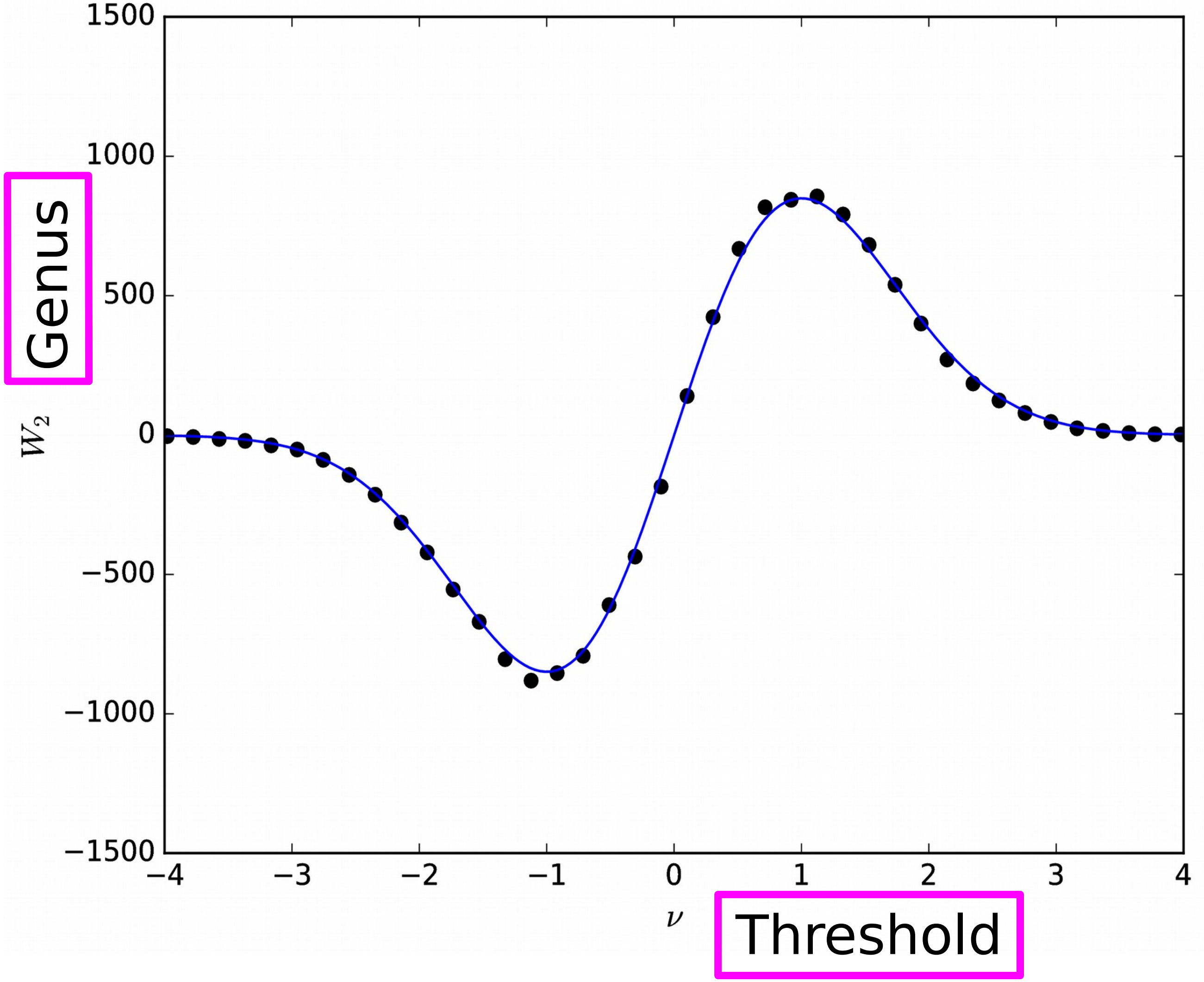
Apply a threshold of constant density

$\nu = 0$



Genus of a Two-Dimensional Field

Genus = number of connected regions – number of holes



Genus - Information Content

KOREA INSTITUTE FOR ADVANCED STUDY

- For a Gaussian field the genus curve shape is fixed, only the amplitude carries information

$$g_{2\mathrm{D}}(\nu) = \frac{1}{2(2\pi)^{3/2}}\frac{\sigma_1^2}{\sigma_0^2}\nu\exp[-\nu^2/2],$$

Adler, 1981

Amplitude: $\frac{1}{2(2\pi)^{3/2}}\frac{\sigma_1^2}{\sigma_0^2}$

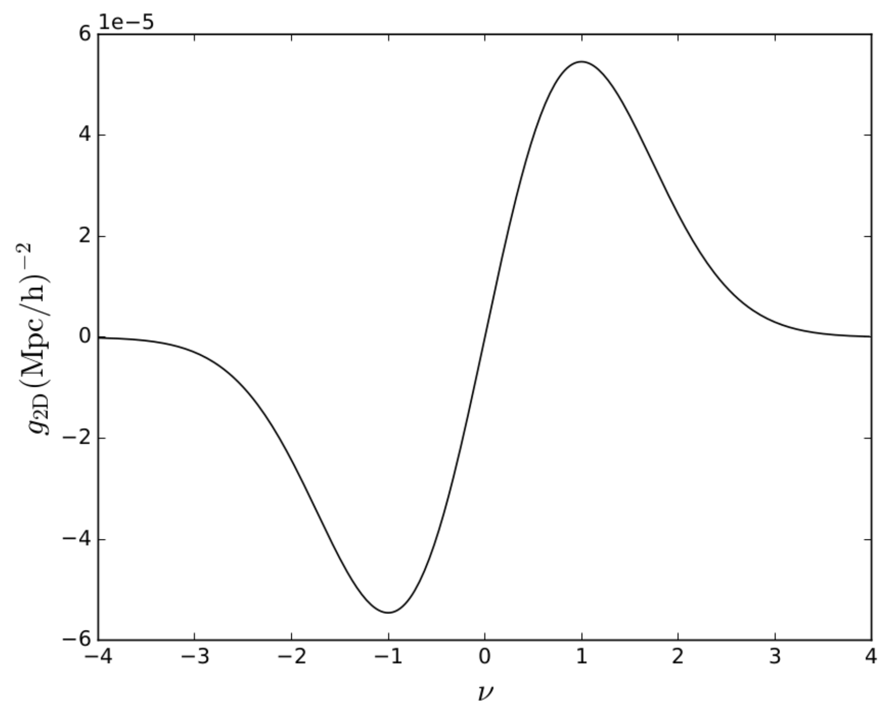
Shape: $\nu\exp[-\nu^2/2]$

$$\sigma_0^2 = \langle \delta_{2\mathrm{D}}^2 \rangle,$$
$$\sigma_1^2 = \langle |\nabla \delta_{2\mathrm{D}}|^2 \rangle$$

$$\sigma_0^2 = \int d^2k_\perp e^{-k_\perp^2 R_\mathrm{G}^2/2} \int dk_3 P_{3\mathrm{D}}(|\vec{k}_\perp + k_3|)\frac{\sin^2[k_3\Delta]}{(k_3\Delta)^2}$$

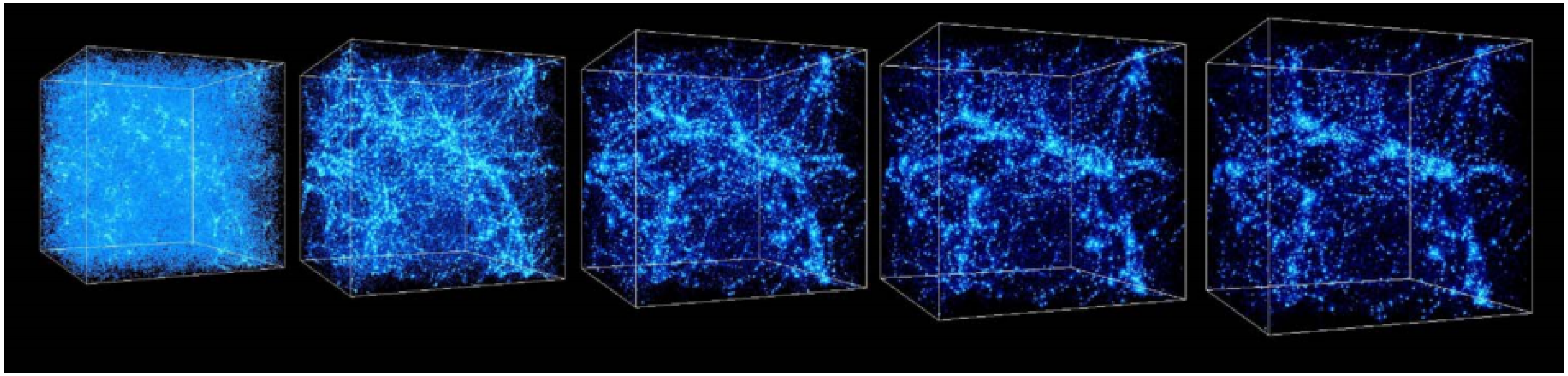
$$\sigma_1^2 = \int d^2k_\perp k_\perp^2 e^{-k_\perp^2 R_\mathrm{G}^2/2} \int dk_3 P_{3\mathrm{D}}(|\vec{k}_\perp + k_3|)\frac{\sin^2[k_3\Delta]}{(k_3\Delta)^2}$$

- As the genus amplitude is a ratio of cumulants, it is insensitive (in principle) to the linear bias and the linear growth factor!

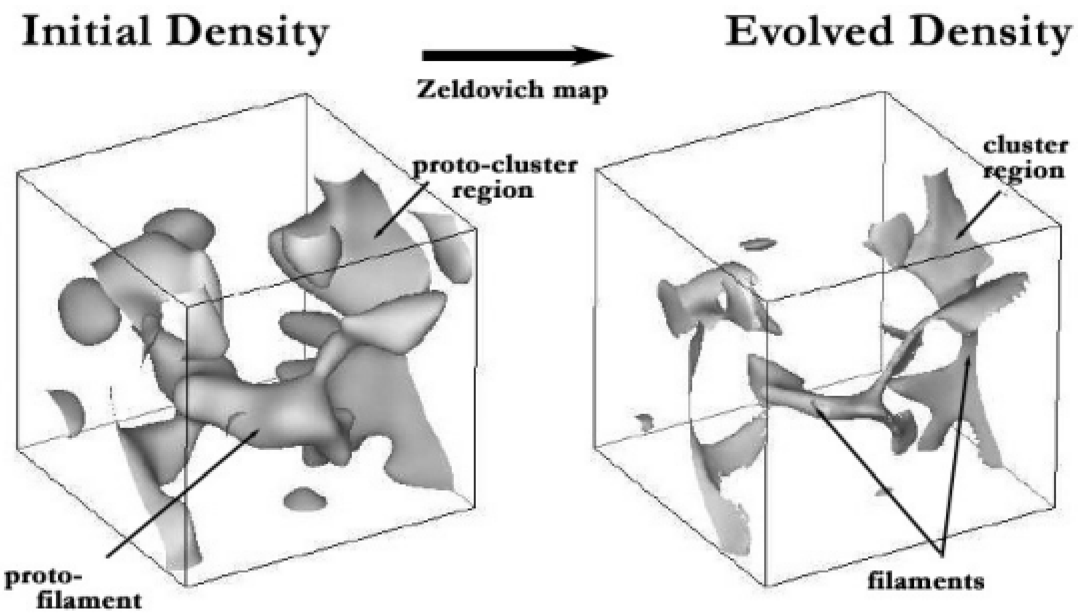


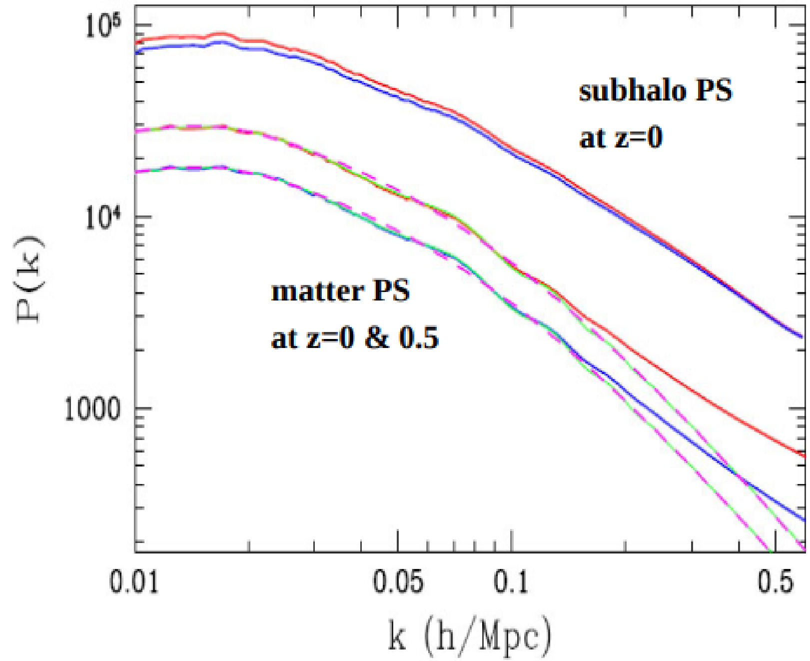
Genus - Information Content

- The topology of the density field can act as a standard population



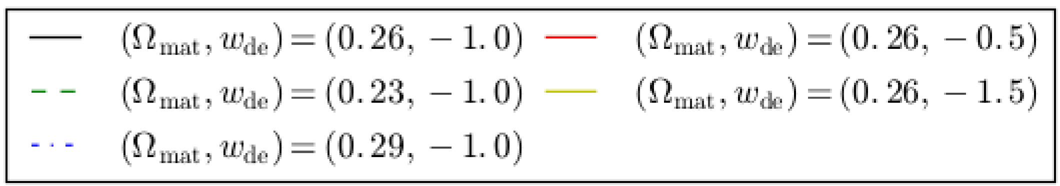
(Visualizations by Andrey Kravtsov)

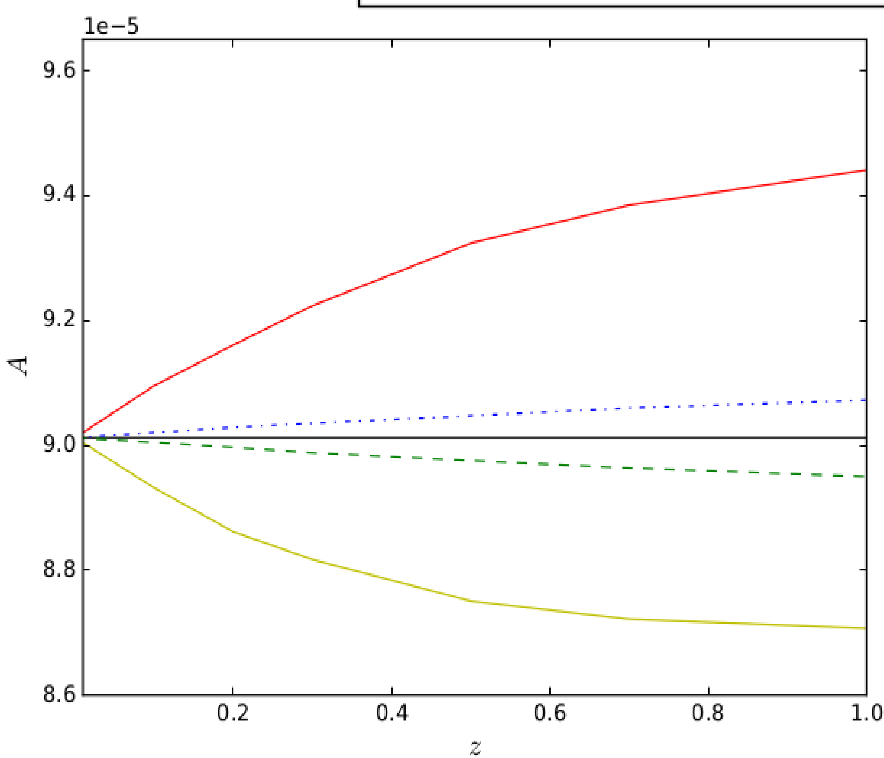


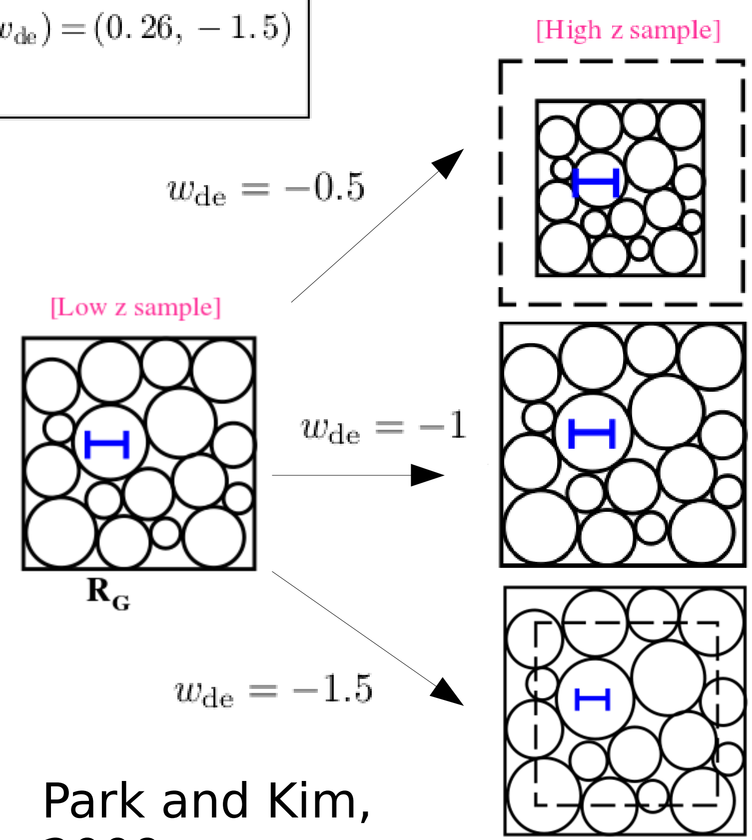


Genus - Information Content

- The genus amplitude carries cosmological information.
- When we smooth the density field over large scales, the genus amplitude is a conserved quantity. We can use this information for cosmological parameter estimation.





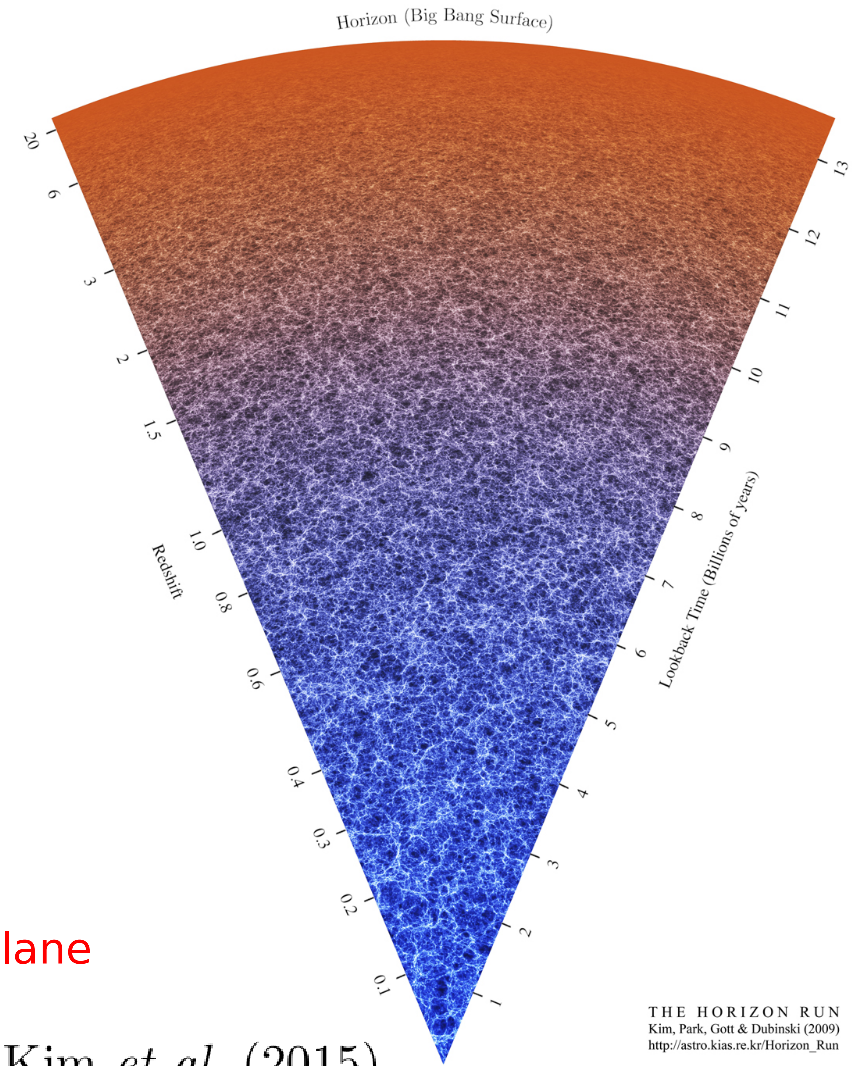


Park and Kim, 2009

Two-Dimensional Genus – Data

KIAS KOREA INSTITUTE FOR ADVANCED STUDY

- We use mock galaxy data from Horizon Run 4 – the latest data release of the Horizon Run Project.
  - $(3150\ \mathrm{Mpc/h})^3$ Box, 6300^3 Particles
- All-sky lightcone data in the redshift range $0 < z < 1$ is taken as our mock data set.
- We apply mass cuts at different redshifts to ensure a constant galaxy number density of $n = 10^{-3}\ (\mathrm{Mpc/h})^{-3}$
- We take slices/shells of constant comoving thickness and calculate the genus of the two-dimensional field



Slice Thickness

$$\Delta = 60\mathrm{Mpc/h}$$

Gaussian Smoothing in the Plane

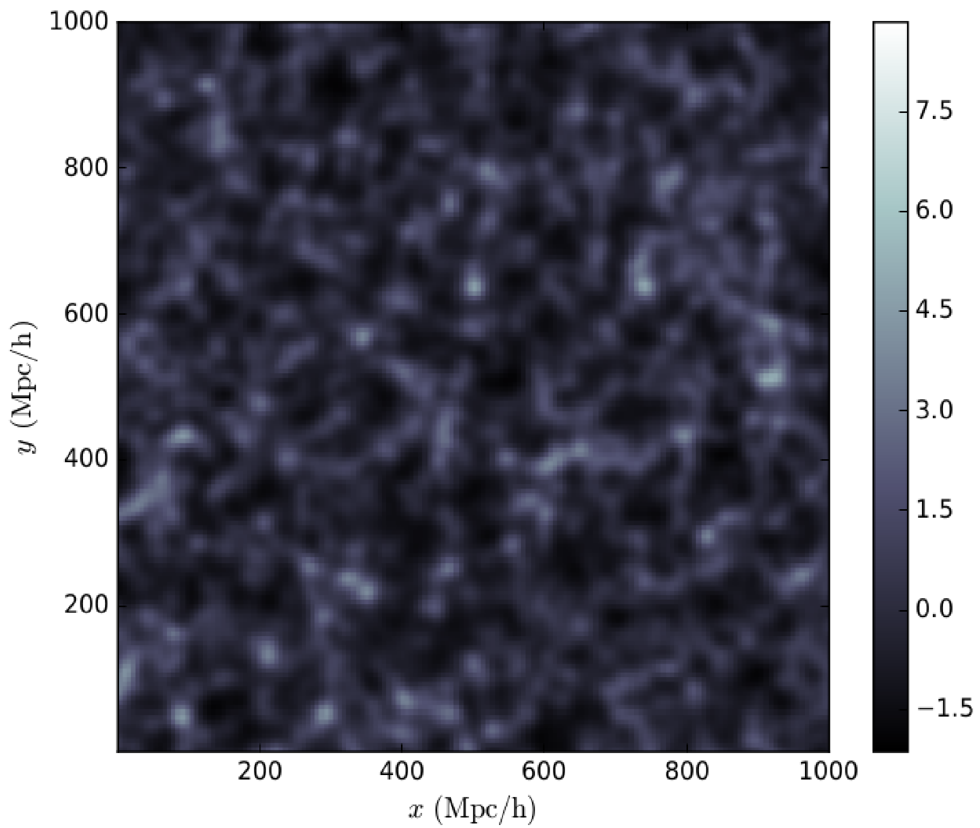
$$R_{\mathrm{G}} = 15\mathrm{Mpc/h}$$

Kim *et al.* (2015)

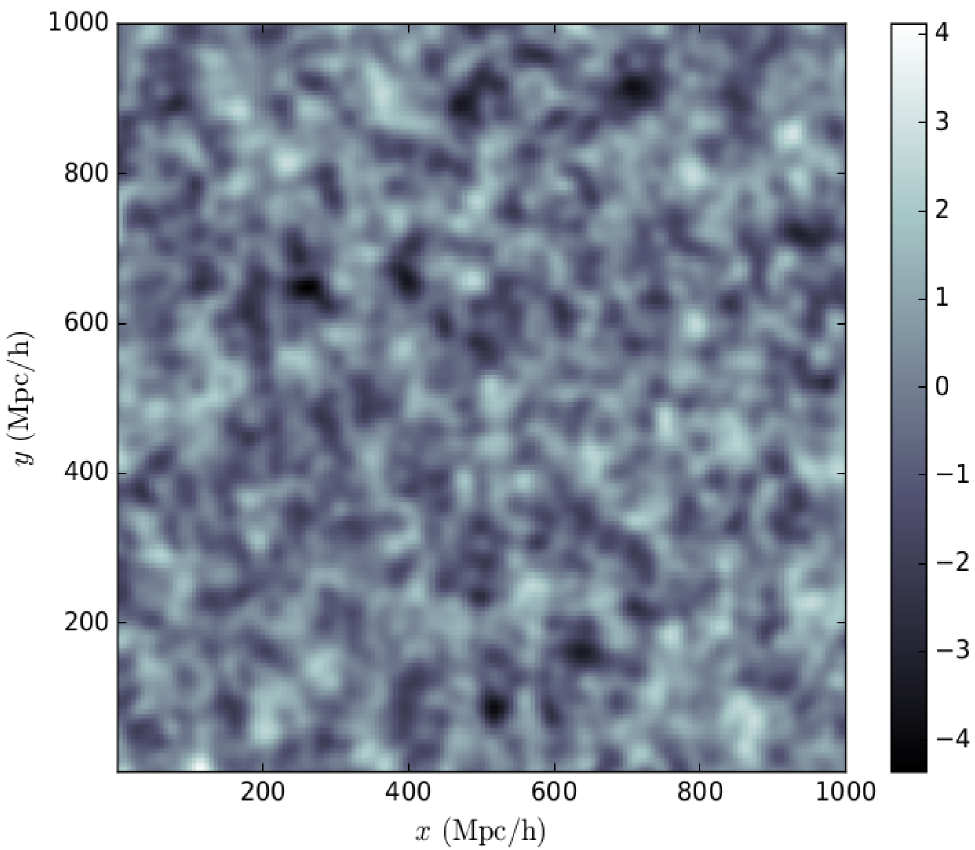
Mock Data

- We wish to extract cosmological information from the amplitude of the genus. N-body simulations are used to study how the genus is modified by gravitational dynamics. We use Horizon Run 4 mock galaxy data, the latest KIAS cosmological scale N-body simulation

Mock Galaxy Catalog



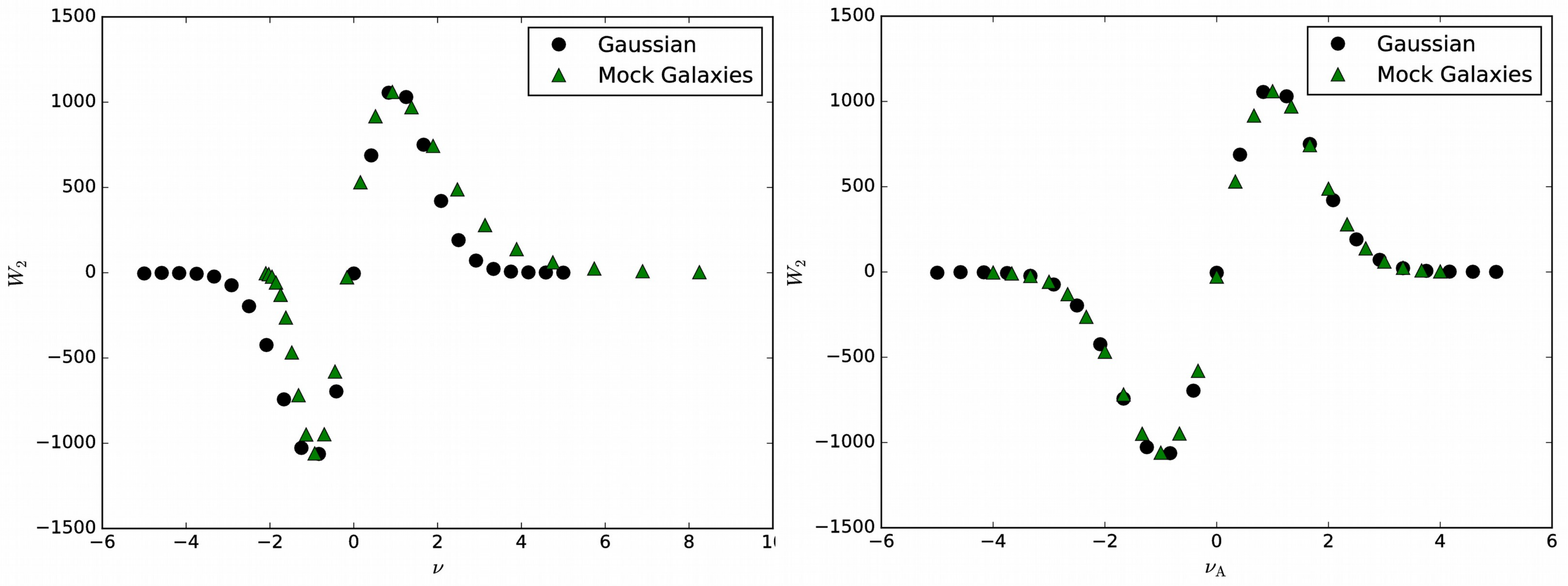
Gaussian



Mock Data

- The genus curve is now modified compared to the Gaussian case.
- The majority of the effect is in the one-point function, which evolves from Gaussian to log-normal
- The amplitude remains almost unaffected by gravitational collapse.

$$f_A = \frac{1}{\sqrt{2\pi}} \int_{\nu_A}^{\infty} \exp[-t^2/2]dt$$

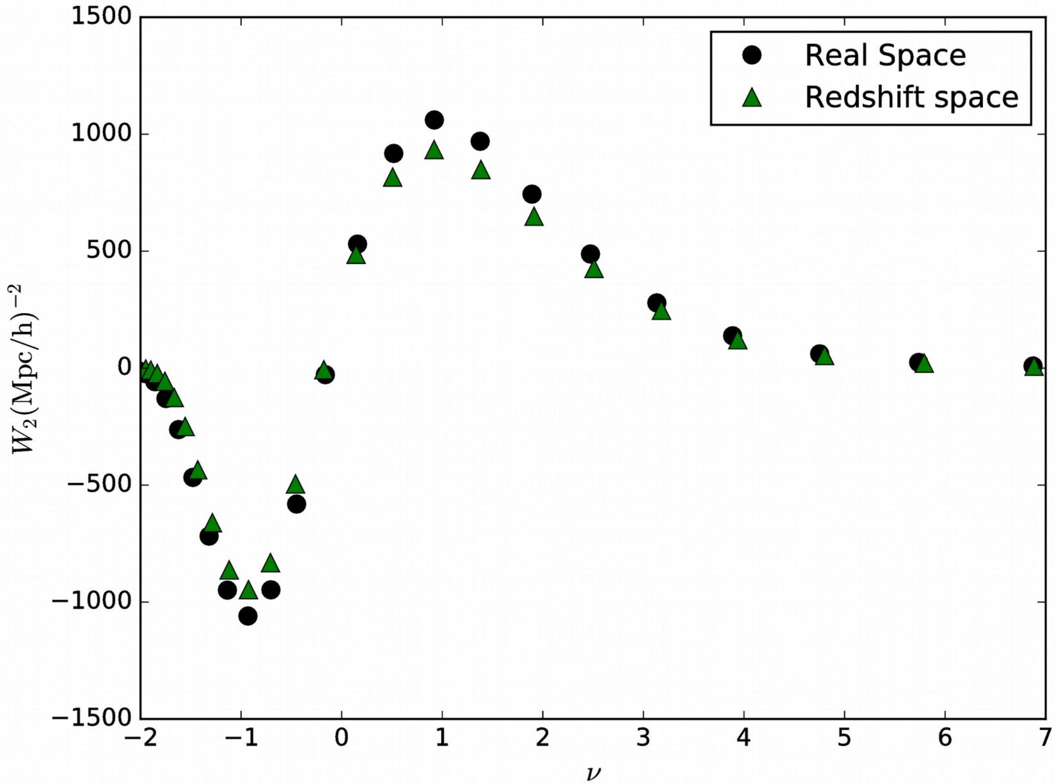


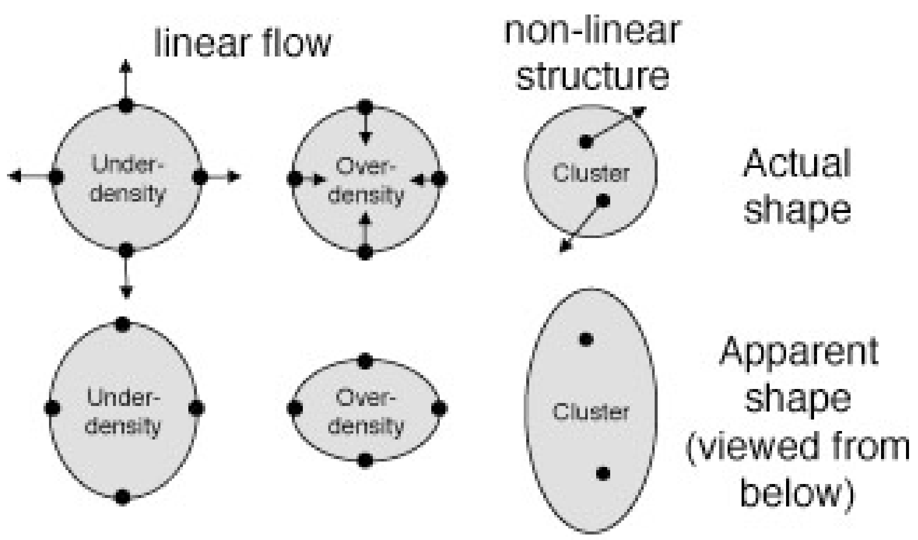
Systematics - RSD

Redshift Space Distortion $g_{\rm 2D}^{\rm RSD}(\nu, \theta_{\rm s}) = a_{\rm RSD}^{(2D)} g_{\rm 2D}^{\rm real}(\nu)$ Matsubara (1996)

$$a_{\rm RSD}^{(2D)} = \frac{3}{2}\sqrt{\left(1-\frac{C_1}{C_0}\right)\left[1-\frac{C_1}{C_0}+\left(\frac{3C_1}{C_0}-1\right)\cos^2(\theta_{\rm s})\right]} \qquad \frac{C_1}{C_0} = \frac{1}{3}\frac{1+6\beta/5+3\beta^2/7}{1+2\beta/3+\beta^2/5}$$

$\beta = f/b$ $\qquad b(z) = 1.6 + z$

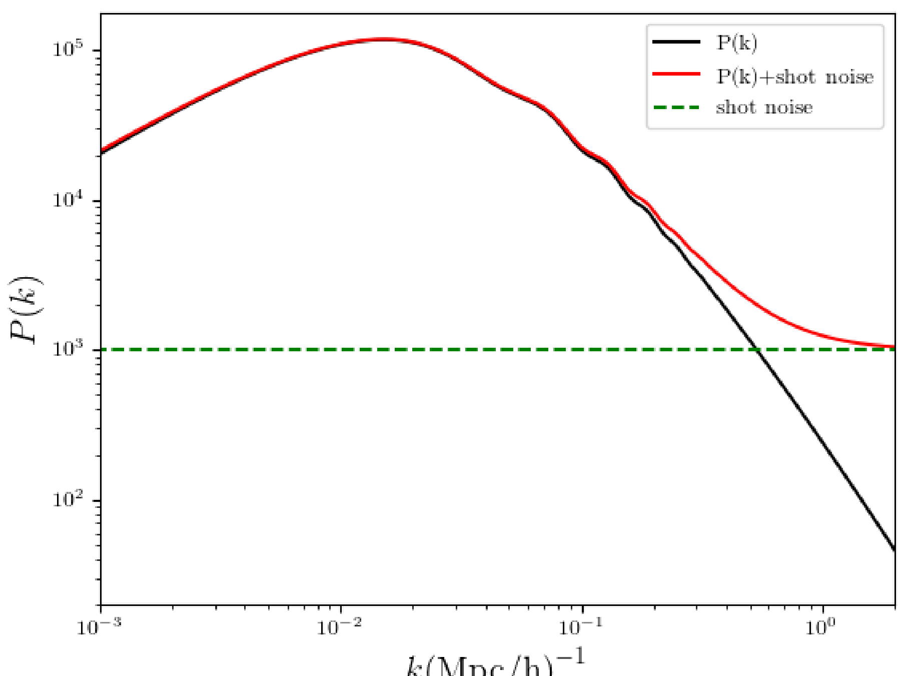


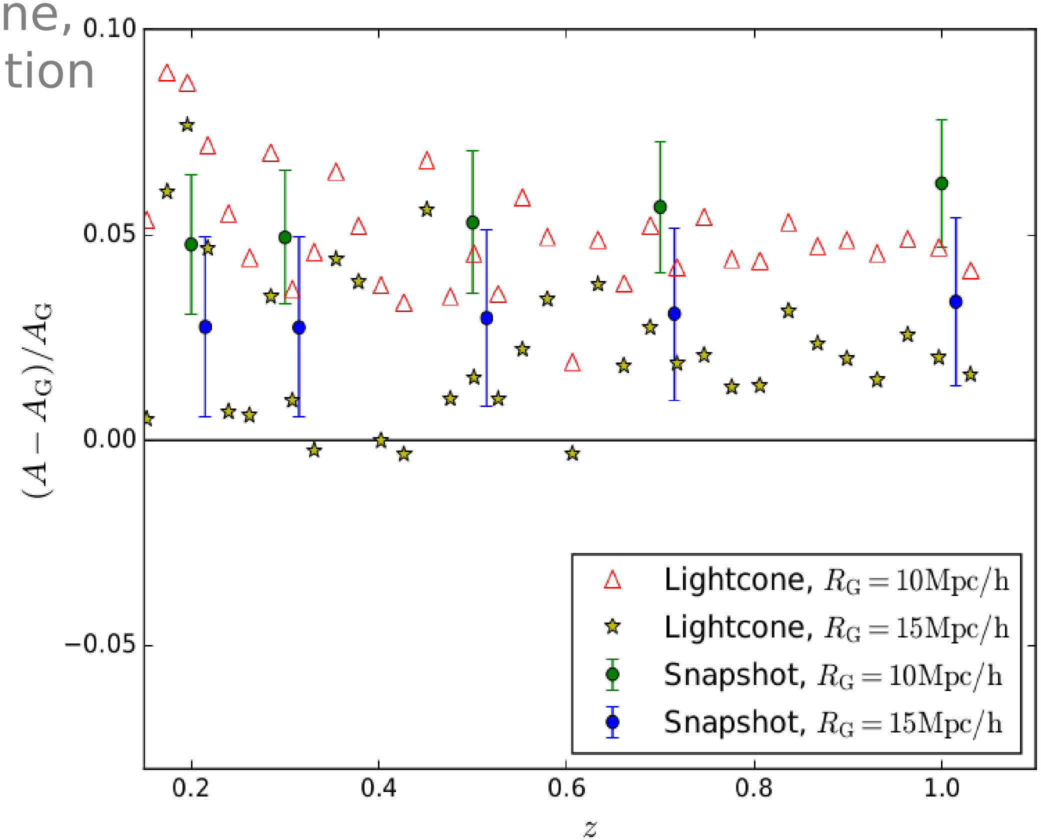


Systematics – Shot Noise

KOREA INSTITUTE FOR ADVANCED STUDY

- Shot Noise modifies the power spectrum by a constant term inversely proportional to the galaxy number density.
- The genus amplitude is related to the integral of the power spectrum over all Fourier modes – small scale physics will affect the statistic!
- We Gaussian smooth the field in the plane, which exponentially suppresses information in the small-k regime.

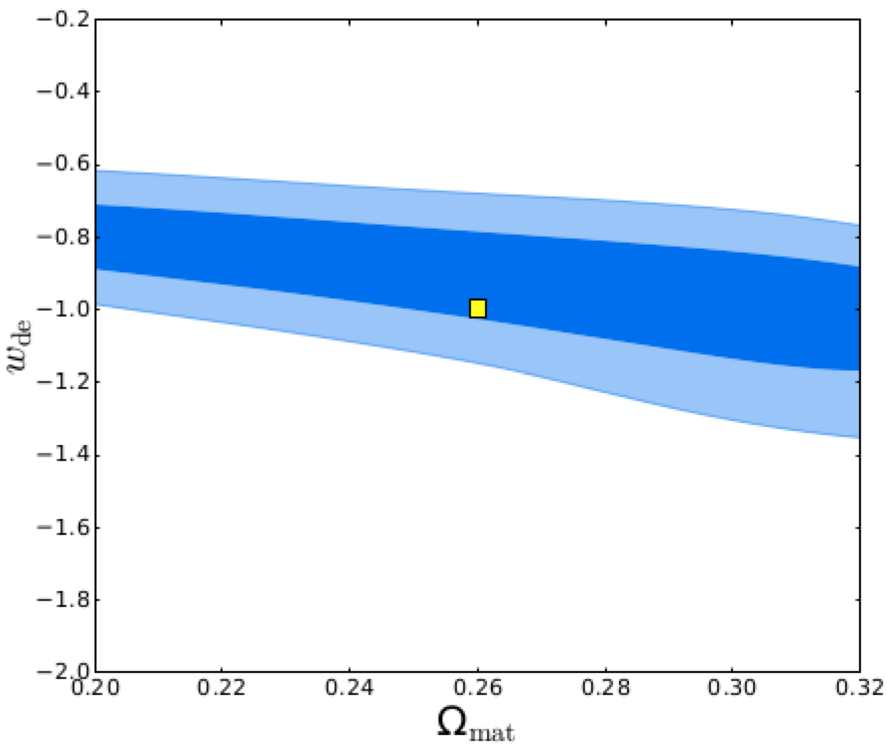


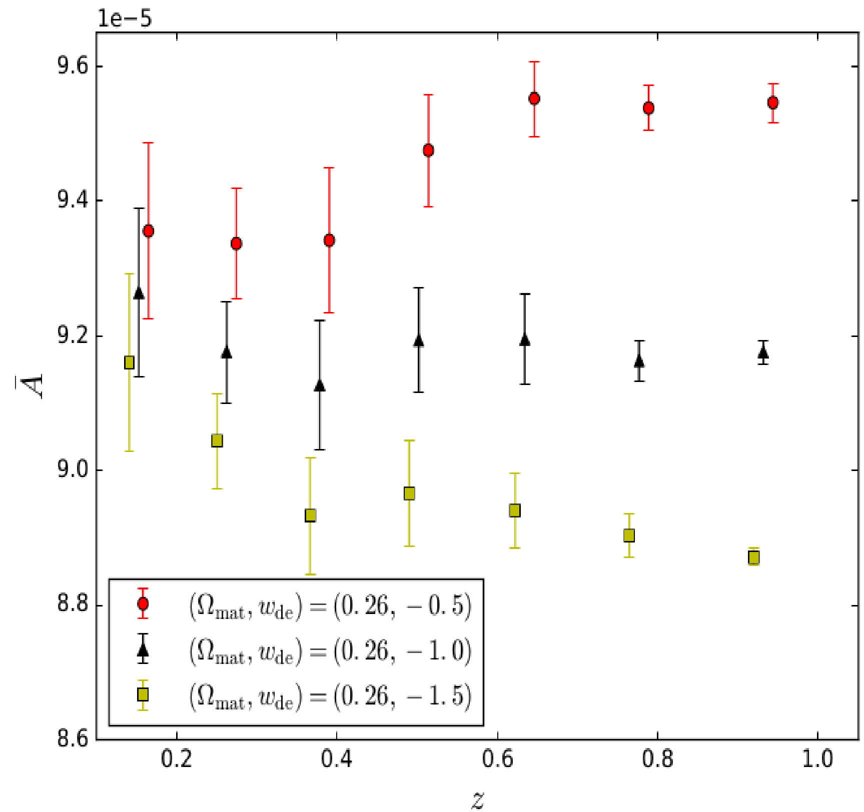


Parameter Constraints - Evolution

$$\chi^2_{\rm evo} = \sum_i \frac{\left(A_i(z_i, \Omega_{\rm mat}, w_{\rm de})/a^{(2D)}_{\rm RSD} - A_0\right)^2}{\sigma_i^2}$$

Parameter	Evo	Mag
$\Omega_{\rm mat}$	$0.262^{+0.081}_{-0.032}$	$0.2616^{+0.009}_{-0.009}$
$w_{\rm de}$	$-0.99^{+0.20}_{-0.16}$	$-1.08^{+0.16}_{-0.30}$

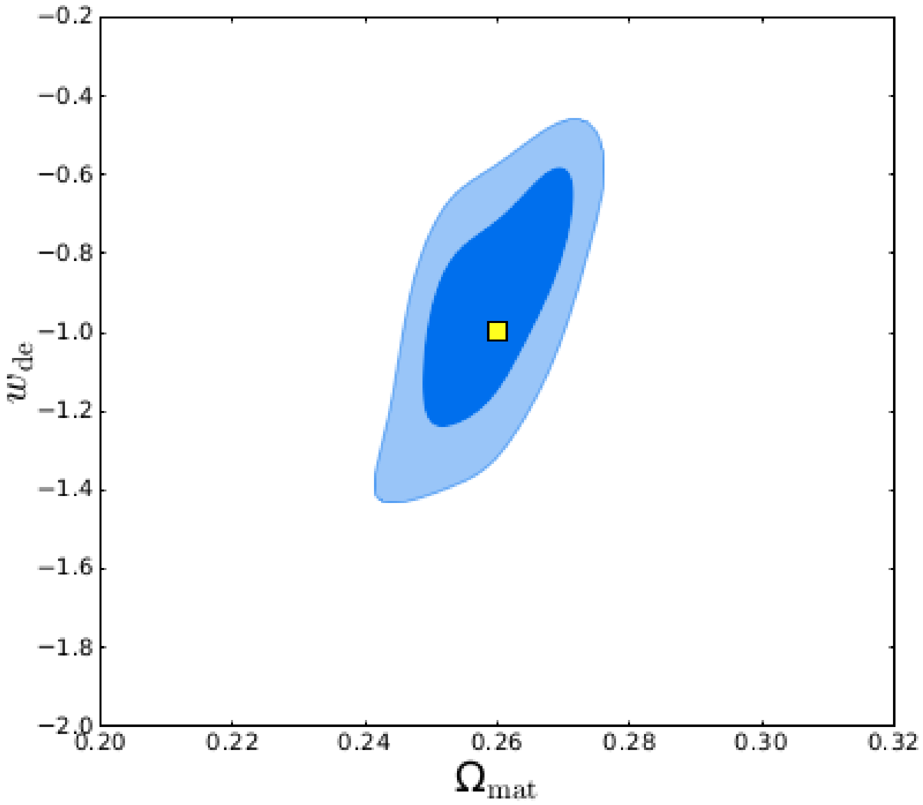


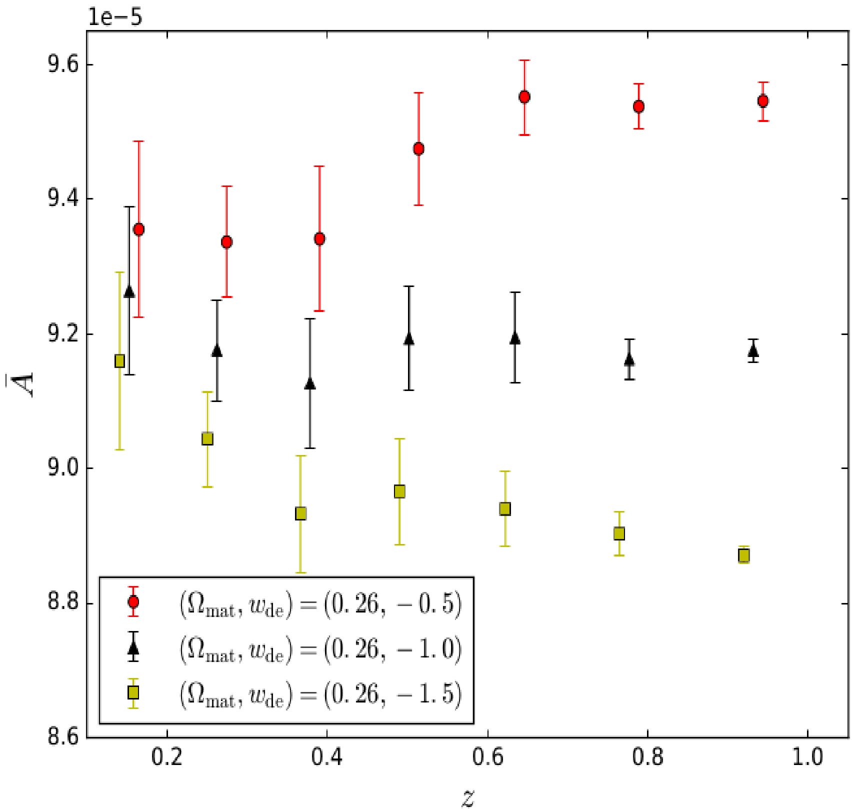


Parameter Constraints

$$\chi^2_{\rm mag} = \sum_i \frac{\left(A_i(z_i,\Omega_{\rm mat},w_{\rm de})(1-\Delta_{\rm SN})/a^{(2D)}_{\rm RSD} - A_{\rm G}(\Omega_{\rm mat},w_{\rm de})\right)^2}{\sigma_i^2+\sigma^2_{\rm RSD}+\sigma^2_{\rm SN}}$$

Parameter	Evo	Mag
$\Omega_{\rm mat}$	$0.262^{+0.081}_{-0.032}$	$0.2616^{+0.009}_{-0.009}$
$w_{\rm de}$	$-0.99^{+0.20}_{-0.16}$	$-1.08^{+0.16}_{-0.30}$





Summary

- We can extract information regarding the initial conditions, composition and evolution of the Universe from the distribution of galaxies for $0 < z < 1$.
- To constrain cosmological parameters, we require dense samples over Gpc volumes. Photometric redshift catalogs are ideal for this purpose.
- If we generate two dimensional fields perpendicular to the line of sight, we do not require accurate redshift information.
- The generation of statistics that can extract information from the point distribution is an open field of research.
- The genus is relatively insensitive to gravitational collapse, galaxy bias and the growth rate of perturbations.
- We are now in a position to apply these statistics to galaxy catalogs!